Quantitative Economics, Trinity Term 2015, Quiz, Week 1

Exercise 1. Using the expression for conditional probability, or otherwise, show that if events A and B are independent, then $P(A|B) = P(A)$.

Proof. Suppose A and B are independent events in S. Then it is true that $P(A \cap B) = P(A)P(B)$. Using the expression for conditional probability we thus have that

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$

□

Exercise 2. Write down the definition of "partition" and state the Total Probability Theorem.

Definition. A partition of a sample space S is a collection $\{A_i, \ldots, A_n\}$ of disjoint events (so that $A_i \cap A_j = \emptyset$for $i \neq j$) with union $\bigcup_{i=1}^{n} A_i = S$

Theorem. *If $\{A_i, \ldots, A_n\}$ is a partition of A such that $P(A_i) > 0$ for all i, then for an event $B \in S$*

$$P(B) = \sum_{i=1}^{n} P(B \cap A_i) = \sum_{i=1}^{n} P(B|A_i)\, P(A_i)$$

Exercise 3. Write down the Bayes formula for conditional probability $P(A_i|B)$. Express this in terms of the Total Probability Theorem.

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(A)} = \frac{P(B|A_i)\, P(A_i)}{\sum_{i=1}^{n} P(B|A_i)\, P(A_i)}$$

Exercise 4. Let X be a continuous random variable with density $f_X(x)$ over the domain $x \in \mathbb{R}$. Write down the expression for expectation and mean of X.

$$\begin{aligned} \mathrm{E}(X) &= \int_{-\infty}^{\infty} x f_X(x) dx \\ \mathrm{Var}(X) &= \int_{-\infty}^{\infty} (x - \mathrm{E}(X))^2 f_X(x) dx \end{aligned}$$

Exercise 5. Write down the general form of the De Morgan's laws for n sets, A_i, $i = 1, \ldots, n$. Can you illustrate these using Venn diagrams with three sets?

$$\begin{aligned} \left(\bigcup_{i=1}^{n} A_i \right)^c &= \bigcap_{i=1}^{n} A_i^c \\ \left(\bigcap_{i=1}^{n} A_i \right)^c &= \bigcup_{i=1}^{n} A_i^c \end{aligned}$$