Econometrics, Michaelmas Term 2015, Quiz 7

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1. Write down the definition of cointegration.

Definition. The components of the vector x_t are said to be co-integrated of order d, b, denoted $x_t \sim CI(d,b)$, if (i) all components of x_t are $I(d)$; (ii) there exists a vector α ($\neq 0$) so that $z_t = \alpha' x_t \sim I(d-b)$, $b > 0$. The vector α is called the cointegrating vector.

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2. How would you test for cointegration in a relationship between two variables, y_t and x_t, that are both non-stationary (i.e. describe the Engle and Granger test for cointegration).

Remark. First, estimate the static long-run relationship $y_t = \beta x_t + z_t$ using OLS and obtain residuals, $\hat{z}_t$. The test for cointegration is then equivalent to testing for a unit root in $\hat{z}_t$, so conduct a Dickey-Fuller test:

$$\begin{aligned} \Delta \hat{z}_t &= \theta \hat{z}_{t-1} + u_t \\ H_0 : \theta &= 0 \ \ (\hat{z}_t \sim I(1)) \\ H_1 : \theta &< 1 \ \ (\hat{z}_t \sim I(0)) \end{aligned}$$

Note that the critical values will not be the same as the standard DF-test. Have to use the MacKinnon critical values.

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3. Write down the general form of the $ARDL(r,p)$ model for variables y_t and x_t, its representation using lag polynomials, including the compact form, and the corresponding assumption for the error term:

$$\begin{aligned} y_t &= \mu + \gamma_1 y_{t-1} + \cdots + \gamma_p y_{t-p} + \beta_0 x_t + \beta_1 x_{t-1} + \cdots + \beta_r x_{t-r} + \varepsilon_t \\ (1 - \gamma_1 L - \cdots - \gamma_p L^p)\, y_t &= \mu + (\beta_0 + \beta_1 L + \cdots + \beta_r L^r)\, x_t + \varepsilon_t \\ C_p(L)\, y_t &= \mu + B_r(L)\, x_t + \varepsilon_t \end{aligned}$$

where $\mathrm{E}[\varepsilon_t | y_{t-1}, \ldots, y_{t-p}, x_t, x_{t-1}, \ldots, x_{t-r}] = 0$.

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4. Using your answer to question (4), write down the $D(L)$ representation of the $ARDL(r,p)$ model. How would you compute (1) impact multiplier; (2) total multiplier; and (3) mean lag.

$$\begin{aligned} y_t &= \frac{\mu}{C_p(L)} + \frac{B_r(L)}{C_p(L)} x_t + \frac{\varepsilon_t}{C_p(L)} \\ y_t &= \alpha + D(L)\, x_t + u_t \\ y_t &= \alpha + \sum_{j=0}^{\infty} \delta_j x_{t-j} + u_t \end{aligned}$$

1. Impact multiplier: $m_0 = \frac{\partial y_t}{\partial x_t} = D(0) = \delta_0 = \frac{B_r(0)}{C_p(0)}$
2. Total multiplier: $m_T = D(1) = \sum_{j=0}^{\infty} \delta_j = \frac{B_r(1)}{C_p(1)}$
3. MeanLag $= \frac{\sum_{j=0}^{\infty} j\delta_j}{\sum_{j=0}^{\infty} \delta_j} = \frac{D'(1)}{D(1)} = \frac{B'(1)}{B(1)} - \frac{C'(1)}{C(1)}$