Econometrics, Michaelmas Term 2015, Quiz 5

Exercise 1. Write down a general form of the ARMA(p,q) process with a non-zero intercept. Re-write this using the AR and MA polynomials in L (lag operator).

$$\begin{aligned} X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} &= \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \\ \left(1 - \phi_1 L - \cdots - \phi_p L^p\right) X_t &= \mu + \left(1 + \theta_1 L + \cdots + \theta_q L^q\right) \varepsilon_t \\ \Phi_p(L) X_t &= \mu + \Theta_q(L) \varepsilon_t \end{aligned}$$

Exercise 2. (1) Write down the definition of a stable ARMA process X_t. (2) Write down the definition of an invertible ARMA process X_t.

1. The ARMA process X_t is stable if the roots of the AR polynomial $\Phi_p(L)$ are outside the unit cirle.
2. The ARMA process X_t is invertible if the roots of the MA polynomial $\Theta_q(L)$ are outside the unit circle.

Exercise 3. Derive conditions for stability and invertibility of the ARMA(1,1) process.

$$\text{ARMA(1,1)}: \quad X_t - \phi_1 X_{t-1} = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

Stable when the roots of $\Phi_p(L)$ are outside the unit cirle ($|L| > 1$) i.e. $|\phi_1| < 1$. Invertible when the roots of $\Theta_q(L)$ are outside the unit circle, i.e. $|\theta_1| < 1$.

Exercise 4. Derive the $MA(\infty)$ respresentation of an AR(1) process $X_t - \phi X_{t-1} = \mu + \varepsilon_t$.

$$\begin{aligned} (1 - \phi L) X_t &= \mu + \varepsilon_t \\ X_t &= (1 - \phi L)^{-1} (\mu + \varepsilon_t) \\ &= \sum_{j=0}^{\infty} \phi^j \varepsilon_{t-j} + \mu (1 - \phi)^{-1} \end{aligned}$$

Exercise 5. Write down the auxiliary regression, null and alterntive hypothesis for the Dickey-Fuller test for a unit root. How would you pick the appropriate deterministic components in the auxiliary regression specification?

$y_t = \rho y_{t-1} + u_t$	$\Delta y_t = \theta y_{t-1} + u_t$
$H_0 : \rho = 1 \quad (y_t \sim I(1))$	$H_0 : \theta = 0 \quad (y_t \sim I(1))$
$H_1 : \rho < 1 \quad (y_t \sim I(0))$	$H_1 : \theta < 1 \quad (y_t \sim I(0))$

Possible deterministic components:

1. $\Delta y_t = \theta y_{t-1} + u_t$.
2. $\Delta y_t = \alpha + \theta y_{t-1} + u_t$.
3. $\Delta y_t = \alpha + \beta t + \theta y_{t-1} + u_t$.