Econometrics, Michaelmas Term 2015, Quiz 4(2)

Exercise 1. Consider a time-series process X_t. Write down the expressions defining the autocovariance function $\gamma(h)$ and the autocorrelation function $\rho(h)$ for this process, where h is the time lag.

$$\begin{aligned}\gamma(h) &= \operatorname{Cov}(X_t, X_{t-h}) = \mathrm{E}\left[(X_t - \mathrm{E}[X_t])(X_{t-h} - \mathrm{E}[X_{t-h}])\right] \\ \rho(h) &= \operatorname{Corr}(X_t, X_{t-h}) = \frac{\operatorname{Cov}(X_t, X_{t-h})}{\sqrt{\operatorname{Var}(X_t)}\sqrt{\operatorname{Var}(X_{t-h})}}\end{aligned}$$

Exercise 2. Write down the definition for a strictly stationary time-series.
A time-series process $\{X_t\}_{t\in\mathbb{Z}}$ is said to be strictly stationary if the joint distribution of $(X_{t_{1,\ldots}} X_{t_k})'$ and $(X_{t_{1+h},\ldots} X_{t_{k+h}})'$ are the same for all positive integers k and for all $t_1, \ldots, t_k$, $h \in \mathbb{Z}$.

Exercise 3. Write down the definition for a weakly stationarity time-series.
A time-series process $\{X_t\}_{t\in\mathbb{Z}}$ is said to be weakly stationary if:

1. $\mathrm{E}[X_t] = m$, for all t.
2. $\mathrm{E}\left[X_t^2\right] < \infty$, for all t.
3. $\operatorname{Cov}(X_t, X_s) = \operatorname{Cov}(X_{t+h}, X_{s+h})$, for all t, s, h.

Exercise 4. What is the difference between a weakly stationary and a strictly stationary time-series? Does any of the imply the other? Is the converse true?
Strict stationarity requires that the whole joint distribution does not change with time, whereas weak stationarity only requires the first two moments to exist and be independent of time. If the first two moments exist, a strictly stationarity ergodic process is also weak stationarity.

Exercise 5. Consider a time-series with a linear deterministic trend $x_t = \mu + \beta t + u_t$, where μ and β are constants, and u_t are *i.i.d.* $(0, \sigma^2)$. Derive expressions for $\mathrm{E}[x_t]$, $\operatorname{Var}[x_t]$ and $\operatorname{Cov}[x_t, x_s]$.

$$\begin{aligned}\mathrm{E}[x_t] &= \mathrm{E}[\mu + \beta t + u_t] = \mathrm{E}[\mu] + \mathrm{E}[\beta t] + \mathrm{E}[u_t] = \mu + \beta t \\ \operatorname{Var}[x_t] &= \operatorname{Var}[\mu + \beta t + u_t] = \operatorname{Var}[\mu + \beta t] + \operatorname{Var}[u_t] + 2\operatorname{Cov}[\mu + \beta t, u_t] = \operatorname{Var}[u_t] = \sigma^2 \\ \operatorname{Cov}[x_t, x_s] &= \operatorname{Cov}[\mu + \beta t + u_t, \mu + \beta s + u_s] \\ &= \operatorname{Cov}[\mu, \mu] + \operatorname{Cov}[\mu, \beta s] + \operatorname{Cov}[\mu, u_s] + \ldots \\ &= 0\end{aligned}$$