Econometrics, Michaelmas Term 2015, Quiz 1

Exercise 1. Suppose $h(x)$ is some predictor of y, where x and y are two variables. Write down the optimization problem for finding the optimal $h(x)$ based quadratic loss function.

$$\min_{h(x)} E\left[(y-h(x))^2 |x\right]$$

Exercise 2. Suppose X and Y are two continuous random variables and $g(Y)$ is some real-valued function of Y. Write down the expression for the conditional expectation of $g(Y)$ given $x \in B$. **Clearly define all notation you introduce.**

Let $f(y|x \in B)$ be the conditional density function of Y given $x \in B$. Then the conditional expectation is given by:

$$E[g(Y)|x \in B] = \int_{-\infty}^{\infty} g(Y) f(y|x \in B)\, dy$$

Exercise 3. Explain (use a formula) what it means for the conditional expectation to be linear operator. **Clearly define all notation you introduce.**
Let x and y be two random variables and $a(x)$ and $b(x)$ two functions of x. Then

$$E[a(x)y + b(x)|x] = E[a(x)y|x] + E[b(x)|x] = a(x)E[y|x] + b(x)$$

Exercise 4. Write down the law of iterated expectations for the simple and the general cases. **Clearly define all notation you introduce.**
Let x, y and w be random variables and $x = f(w)$ for some non-stochastic function $f(\cdot)$. Then the Law of Iterated Expectations can be expressed as follows:

$$\begin{aligned} E[y] &= E[E[y|x]] \\ E[y|x] &= E[E[y|w]|x] \end{aligned}$$

Exercise 5. Suppose that you want to estimate a linear relationship between two observed random variables y_i and x_i. Write down the linear regression model (with intercept), the Objective Function for the Ordinary Least Squares and the First Order Conditions (FOCs). **Clearly define all notation you introduce.**
The linear model is given by $y_i = \beta_1 + \beta_2 x_i + u_i$, where β_1 and β_2 are parameters to be estimated and u_i is the stochastic error term (residual). We want to find $b = (b_1, b_2) \in \mathbb{R}^2$ that minimizes the Sum of Squared Residuals. The objective function, L, is then given by:

$$L = \min_{b \in \mathbb{R}^2} \sum_{i=1}^{n} u_i^2 = \min_{b \in \mathbb{R}^2} \sum_{i=1}^{n} (y_i - b_1 - b_2 x_i)^2$$

where n is the number of observations. Differentiate with respect to b_1 and b_2 respectively to obtain the FOCs:

$$\begin{aligned} \frac{\partial L}{\partial b_1} &= -2\sum_{i=1}^{n}(y_i - b_1 - b_2 x_i) = 0 \\ \frac{\partial L}{\partial b_2} &= -2\sum_{i=1}^{n}(y_i - b_1 - b_2 x_i)\, x_i = 0 \end{aligned}$$